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Readers' Forum

Comment on "Modal Analysis of Structures with Holonomic Constraints"

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IN the paper under consideration Yang[1] studied the influence of the introduction of additional constraints in damped gyroscopic mechanical systems. In particular, the author discussed the influence of the constraints on the system's stability behavior. Several theorems were formulated and interesting results were obtained. Unfortunately the term stable system is not clearly defined in the paper, and it is used with different meanings in different contexts.

One of the author's conclusions is that "for a stable, initially unconstrained, free vibration system, if the added constraints is scleronomic, the constraint system remains stable." Here is a simple counterexample that we borrow from Ref. 2. Consider the point mass elastically supported in a rigid ring rotating with circular frequency Ω according to Fig. 1a. Its equations of motion are

$$m\ddot{x} + (c - \Omega^2 m)x - 2m\Omega\dot{y} = 0 \quad (1)$$

$$m\ddot{y} + (c - \Omega^2 m)y + 2m\Omega\dot{x} = 0 \quad (2)$$

Introducing the new, dimensionless variable $\tau = \Omega t$, and indicating the derivative with respect to τ by a prime leads to

$$x'' + (\mu^2 - 1)x - 2y' = 0 \quad (3)$$

$$y'' + (\mu^2 - 1)y + 2x' = 0 \quad (4)$$

with $\mu^2 = (c/m)/\Omega^2$. Setting

$$x = \hat{x}e^{s\tau}, \qquad y = \hat{y}e^{s\tau} \quad (5)$$

leads to the characteristic equation

$$s^2 + (2\Delta + 4)s^2 + \Delta^2 = 0 \quad (6)$$

with $\Delta = \mu^2 - 1$. From Eq. (6) it is obvious that both roots s_1^2 and s_2^2 are real and negative as long as $\Delta \neq 0$, i.e., both for $\Omega^2 < c/m$ as well as for $\Omega^2 > c/m$ the trivial solution of Eqs. (1) and (2) is stable in the sense of Lyapunov (not asymptotically stable!).

On the other hand, if the constraint $y = 0$ is introduced in the original system, such that the point mass moves along the x axis according to Fig. 1b, the equation of motion of the reduced system is given by

$$m\ddot{x} + (c - \Omega^2 m)x = 0 \quad (7)$$

The trivial solution is obviously unstable in the sense of Lyapunov for $\Omega^2 > c/m$. Here we, therefore, have an example in which the introduction of a holonomic constraint in a gyroscopic system leads to instability, in contradiction to the theorem formulated in the paper.

Of course, if stability is defined in a different sense, for example as "bounded input/bounded output," then the counterexample given would no longer be valid, since it refers to one with free oscillations. Consider the two-degree-of-freedom system of Fig. 2. Its equations of motion are

$$m\ddot{x}_1 + 2cx_1 - cx_2 + d\dot{x}_1 = f_1(t) \quad (8)$$

$$m\ddot{x}_2 + 2cx_2 - cx_1 = f_2(t) \quad (9)$$

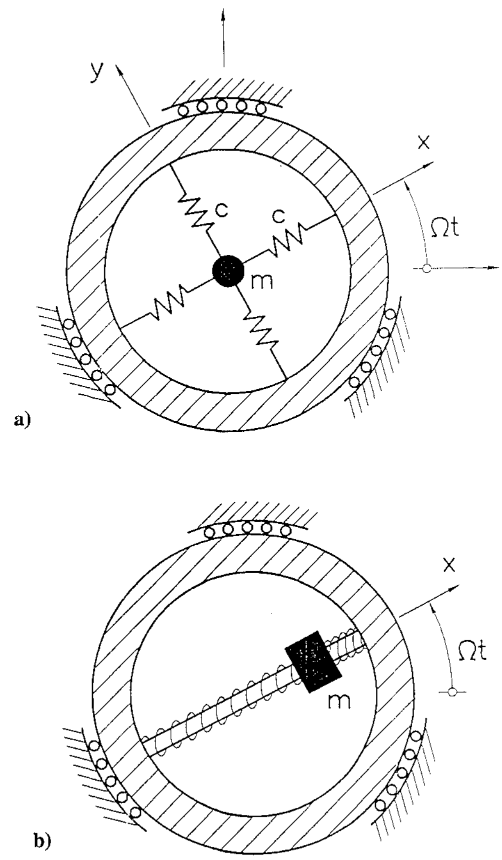

Fig. 1 Simple example with gyroscopic terms.

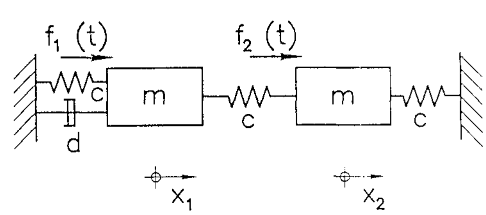

Fig. 2 Forced two-degree-of-freedom system.

The free system, i.e., the system without the external excitation $[f_1(t) = f_2(t) = 0]$ is pervasively damped. This implies that the trivial solution is asymptotically stable. Damping is complete if the damping matrix is positive definite (see Ref. 2); for the definition of pervasiveness see Ref. 4. If the constraint $x_1 = 0$ is introduced, then the resulting new system is obviously undamped and has the natural frequency $\omega = \sqrt{2c/m}$. As a consequence, a forcing term of the type $f_2(t) = \hat{f}\cos\omega t$ will give rise to an unbounded solution in the constrained system. The constrained system, therefore, is not stable in the sense of bounded input/bounded output.

References

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[3]Hagedorn, P., and Otterbein, S., *Technische Schwingungslehre*, Springer–Verlag, Berlin, 1987.

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